The Co-operative University of Kenya

END OF SEMESTER EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE AND BACHELOR OF INFORMATION TECHNOLOGY

COURSE CODE: BMAT 2109

UNIT TITLE: LINEAR ALGEBRA

DATE: DECEMBER, 2019 Time: 2 Hours

INSTRUCTIONS

1. Answer question **ONE** and any other **TWO (2)** questions
2. Scientific Calculators and non-programmable calculators may be used

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1. (a) Solve $\frac{m-2}{3}+1=\frac{2m}{7}$ (**3** *marks*)

   (b) Determine whether the vectors $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$, $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$ and $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ are linearly independent in $\mathbb{R}^3$ (**5** *marks*)

   (c) Find the determinant of the matrix $A=\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix}$ (**2** *marks*)

   (d) Let $A=\begin{bmatrix} 3 & 1 \\ 4 & -5 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 2 \\ -5 & 7 \end{bmatrix}$, find 3A+5B (**3** *marks*)

   (e) Solve the following system by Gaussian elimination

$$\begin{aligned} x-3y-2z&=6 \\ 2x-4y-3z&=8 \\ -3x+6y+8z&=-5 \end{aligned}$$

   (**10** *marks*)

   (f) Define a mapping $T:\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} e^x \\ e^y \end{bmatrix}$
   Determine whether T is a linear transformation. (**4** *marks*)

   (g) Let $u_1=(1,1,0)$, $u_2=(1,1,1)$ and $u_3=(0,2,3)$. Find the volume V(S) of the parallelopiped S in $\mathbb{R}^3$ determined by the three vectors. (**3** *marks*)

2. Let $A=\begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$.

   (a) Write the characteristic polynomial of A. (**2** *marks*)

   (b) Write the characteristic equation of A. (**2** *marks*)

   (c) Find the eigenvalues of A (**4** *marks*)

(d) Find the eigenvectors corresponding to each of the eigenvalues found in part (c). (**10** *marks*)

3. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be the linear transformation defined by

$$T(\mathbf{v}) = T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$$

and let $B = \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}\right\}$ $\quad B' = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$

be ordered bases for $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively

(a) Find the matrix $[T]_B^{B'}$ (**10** *marks*)

(b) Let $\mathbf{v} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$. Find T($\mathbf{v}$) directly and then use the matrix found in part (a). (**10** *marks*)

4. (a) Use Cramer's rule to solve

$$\begin{aligned} x + 2y + z &= 5 \\ 2x + 2y + z &= 6 \\ x + 2y + 3z &= 9 \end{aligned}$$

(**15** *marks*)

(b) calculate the area of the parallelogram determined by the points (-2,-2), (0,3), (4,-1) and (6,4) (**5** *marks*)

5. Diagonalize $A = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$ (**20** *marks*)