

FINANCIAL MATHEMATICS EXAM

Instructions: Answer question one and any other two questions

QUESTION 1

- i. Calculate the present value of an annuity that pays £150 pa annually in arrears forever using an annual effective rate of interest of 8%. [4 marks]
 - ii. Find the present value as at 1 January 2004 of a series of payments of £100 payable on the first day of each month during 2005, 2006 and 2007, assuming an effective rate of interest of 8% per annum. [5 marks]
 - iii. The nominal rate of discount per annum convertible quarterly is 8%.
 - a) Calculate the equivalent effective rate of interest per annum. [3 marks]
 - iv. A company has borrowed £500,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for ten years after a grace period of 3 years from the date the loan is made.
 - v. The annual instalments are calculated at an effective rate of interest of 9% per annum.
 - i. Calculate:
 - a) The amount of the level annual instalments. [3 marks]
 - b) The total amount of interest which will be paid over the ten-year term. [4 marks]
 - vi. At the beginning of the eighth year, immediately after the seventh instalment has been made, the company asks for the loan to be rescheduled over a further four years from that date. The bank agrees to do this on condition that the rate of interest is increased to an effective rate of 12% per annum for the term of the rescheduled instalments and that repayments are made quarterly in arrears.
 - a) Calculate the amount of the new quarterly instalment. [4 marks]
 - b) Calculate the interest content of the second quarterly instalment of the rescheduled loan repayments. [4 marks]
- [Total 30 marks]

QUESTION 2

- i. Discuss the merits of net present value as a project evaluation technique. [5 marks]
- ii. A company is undertaking a new project. The project requires an investment of £5m at the outset, followed by £3m three months later. It is expected that the investment will provide income over a 15 year period starting from the beginning of the third year. Net income from the project will be received continuously at a rate of £1.7m per annum. At the end of this 15 year period there will be no further income from the investment.

Calculate at an effective rate of interest of 10% per annum:

- a) the net present value of the project [5 marks]
- b) the discounted payback period [5 marks]
- iii. A bank has offered to loan the funds required to the company at an effective rate of interest of 10% per annum. Funds will be drawn from the bank when required and the loan can be repaid at any time. Once the loan is paid off, the company can earn interest on funds from the venture at an effective rate of interest of 7% per annum.
 - a) Calculate the accumulated profit at the end of the 17 years. [5 marks]

[Total 20 marks]

QUESTION 3

- i. Differentiate between the following annuities
 - a) Perpetual annuity and certain annuity [2 marks]
 - b) Deferred annuity and immediate annuity [2 marks]
- ii. A loan is repayable by annual instalments paid in arrear for 20 years. The first instalment is £4,650 and each subsequent instalment is £150 greater than the previous instalment.

Calculate the following, using an interest rate of 9% per annum effective:

- a) The amount of the original loan [5 marks]
- b) The capital repayment in the tenth instalment [4 marks]
- c) The interest element in the last instalment [4 marks]
- d) The total interest paid over the whole 20 years [3 marks]

[Total 20]

QUESTION 4

Two investment projects are being considered.

- i. Explain why comparing the two discounted payback periods or comparing the two payback periods are not generally appropriate ways to choose between two investment projects. [4 marks]
- ii. The two projects each involve an initial investment of £3m. The incoming cash flows from the two projects are as follows:

Project A

In the first year, Project A generates cash flows of £0.5m. In the second year it will generate cash flows of £0.55m. The cash flows generated by the project will continue to increase by £0.05m per annum until the end of the sixth year and will then cease.

Assume that all cash flows are received in the middle of the year.

Project B

Project B generates cash flows of £0.64m per annum for six years. Assume that all cash flows are received continuously throughout the year.

(a) Calculate the non-discounted payback period from Project B.

(b) Calculate the discounted payback period from Project B at a rate of interest of 4% per annum effective. [6 marks]

- iii. Show that there is at least one “cross-over point” for Projects A and B between 0% per annum effective and 4% per annum effective where the cross-over point is defined as the rate of interest at which the net present value of the two projects is equal. [6 marks]
- iv. Explain why the net present value of Project A appears to fall more rapidly than the net present value of Project B as the rate of interest increases. [4 marks]

[Total 20 marks]

Question 5

A country’s football association is considering whether to bid to host the World Cup in 2030. Several countries aspiring to host the World Cup will be making bids. Regardless of whether the bid is successful, the association will incur various costs.

For two years, starting on 1 January 2017, the association will incur costs at a rate of £2m per annum, assumed to be paid continuously, to prepare the bid.

If the football association is successful, the following costs will be incurred from 1 January 2020 until 31 December 2029:

- One stadium will be built each year for ten years. The first stadium will be built in 2020 and is expected to cost £200m; the stadium built in 2021 is expected to cost £210m; and so on, with the cost of each stadium rising by £10m each year. The costs of building each stadium are assumed to be incurred halfway through the relevant year.

- Administration costs at a rate of £100m per annum will be incurred, payable monthly in advance from 1 January 2029 until 31 December 2030.

- Revenues from television, ticket receipts, advertising and so on are expected to be £3,300m and are assumed to be received continuously throughout 2030.

Explain why the payback period is not a good indicator of whether this project is worthwhile.

[3 marks]

The football association decides to judge whether to go ahead with the bid by calculating the net present value of the costs and revenues from a successful bid on 1 January 2017 at a rate of interest of 4% per annum effective.

Determine whether the association should make the bid. [13 marks]

The football association is discussing how it might factor into its calculations the fact that it is not certain to win the right to host the World Cup because other countries are also bidding.

Explain how you might adjust the above calculations if the probability of winning the right to host the World Cup is 0.1 and whether this adjustment would make it more likely or less likely that the bid will go ahead.

[4 marks]

[Total 20]