The Co-operative University of Kenya

END OF SEMESTER EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN DISASTER RISK MANAGEMENT AND SUSTAINABLE DEVELOPMENT

COURSE CODE: BSTA 2104

UNIT TITLE: PROBABILITY AND STATISTICS II

DATE: DECEMBER, 2019 Time: 2 Hours

INSTRUCTIONS

1. Answer question **ONE** and any other **TWO (2)** questions
2. Show all your workings
3. Scientific Calculators and non-programmable calculators may be used
4. Use the provided statistical tables where applicable

SECTION A: Answer ALL questions (30 MARKS)

1. (a) Define the following terms (**4** *marks*)

   i. Random Variable

   ii. Bernoulli Distribution

   (b) For two discrete random variables X and Y, show that $E(X+Y) = E(X) + E(Y)$ (**3** *marks*)

   (c) Suppose that $Y = -2X + 3$. If we know $E(Y) = 1$ and $E(Y^2) = 9$, find $E(X)$ and $Var(X)$. (**5** *marks*)

   (d) Let $X \sim Geometric(p)$. Find $E(X)$ (**5** *marks*)

   (e) Let X and Y be two independent $Geometric(p)$ random variables. Find the PMF of $Z = X - Y$ (**6** *marks*)

   (f) Let $X \sim N(10, 25)$, Find (**7** *marks*)

   i. $P(X < 8)$

   ii. $P(-1 < X < 12)$

   iii. $P(X > 11 | X > 15)$

SECTION B: Answer any two (2) questions (40 MARKS)

2. (a) Let $X \sim N(\mu_x, \sigma_x^2)$, Find the distribution of $Y = aX + b$ (**3** *marks*)

(b) Let $X \sim Geometric(p)$. Find $E\left(\frac{1}{3^x}\right)$ **(4** *marks*)

(c) Let $X \sim Uniform(a, b)$. Show that the mean and variance of the random variable X are $\frac{a+b}{2}$ and $\frac{(b-a)^2}{12}$ respectively **(6** *marks*)

(d) I toss a biased coin twice. If the probability of observing a head is twice as likely as the tail and X is a random variable defining the number of heads I observe, find the probability mass function X hence or otherwise the variance of X **(7** *marks*)

3. (a) Find the m.g.f of a geometric random variable and use it to obtain the mean and the variance of the random variable **(5** *marks*)

(b) Let $X \sim Gamma(\alpha, \lambda)$, where $\alpha, \lambda > 0$. Find $E(X)$, and $Var(X)$ **(5** *marks*)

(c) Let X be a continuous random variable PDF $f(x) = \begin{cases} kx^4, & 0 < x \leqslant 1 \\ \\ 0, & \text{Otherwise} \end{cases}$

Find, **(10** *marks*)

i. the constant k

ii. the CDF of X

iii. $P(X \leq \frac{3}{5} | X > \frac{1}{2})$

iv. PDF of $Y = \frac{1}{X}$

4. (a) Let X be a discrete random variable with range $R = 1, 2, 3, ...$ Suppose the PMF of X is given by

$$P(x) = \frac{1}{2^x} \text{ for } x = 1, 2, 3, ...$$

Find; **(8** *marks*)

i. the CDF of X, $F(x)$ and plot it

ii. $P(3 < X \leq 7)$

iii. $P(X > 5)$

(b) Consider the table below and answer the questions below **(12** *marks*)

	$X = -1$	$X = 0$	$X = 1$
$Y = -1$	0	$\frac{1}{4}$	$\frac{1}{5}$
$Y = 0$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$
$Y = 1$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$

Find

i. Marginal distributions of X and Y

ii. the conditional probability distribution of X given Y=1

iii. Find $Cov(X, Y)$ and $\rho(X, Y)$

5. (a) State and prove the weak law of large numbers **(4** *marks***)**

   (b) Let X and Y be jointly normal random variables with parameters μ_x, σ_x,μ_y, σ_y, and ρ. Find the conditional distribution of Y given $X = x$ **(6** *marks***)**

   (c) Let X and Y be jointly continuous random variables PDF $f(x, y) = \begin{cases} 1 + cx, & x, y \geqslant 0, \\ \\ 0, & \text{Otherwise} \end{cases}$

   Find, **(10** *marks***)**

   i. the constant c

   ii. the marginal PDFs of X and Y

   iii. $P(Y < 2X^2)$